$$f(x) = \frac{x^2}{2} + \frac{5x^4}{24} + 8x^5 + x^5\varepsilon(x)$$

$$g(x) = \frac{x^5}{120} + x^5\varepsilon(x)$$

$$\frac{f(x)}{g(x)} = \frac{\cancel{x^5}\left(\frac{1}{2x^3} + \frac{5}{24x} + 8 + \varepsilon(x)\right)}{\frac{\cancel{x^5}}{120}} \xrightarrow[x\to 0]{} \frac{1}{15}$$

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{1}{15}.$$

$$\left(\frac{1}{1+x^2}\right)' = \left(-\frac{2x}{(1+x^2)^2}\right)' = \frac{-2(1+x^2)^2 + 2x(4x(1+x^2))}{(1+x^2)^4}$$

$$= \frac{\cancel{(1+x^2)}\left(2(1+x^2) - 8x^2\right)}{(1+x^2)^3}$$

$$= 2$$

<u>6.11</u>

$$2.\ f(x) = \arctan(x) = \sum_{k=0}^{3} \frac{f^{(k)}(0)}{k!} x^k + x^3\varepsilon(x)$$

$$f(x) = x - \frac{x^3}{3}. + x^3\varepsilon(x)$$

$$\boxed{u = \frac{1}{x}} \Rightarrow x = \frac{1}{u}$$

$$\arctan\left(\frac{1}{u}\right) = \frac{1}{u} - \frac{1}{3u^3} + \frac{1}{u^3}\varepsilon\left(\frac{1}{u}\right).$$

$$\arctan(u) = +\frac{\pi}{2} - \frac{1}{u} + \frac{1}{3u^3}.$$

$$\boxed{\arctan\left(\frac{1}{x}\right) = \frac{\pi}{2} - \arctan(x)}.\quad x > 0.$$

$$g(x) = \frac{\ln(x)}{x^2}$$ DL en 1.

$$g(x) = \frac{\ln(1+(t-1))}{(t-1)^2}$$ avec $x = t-1$.

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} + \frac{x^5}{120}$$

$$\frac{\cancel{(t-1)^2}\left(\frac{1}{(t-1)} - 2 + \frac{t-1}{6} - \frac{(t-1)^2}{24} + \frac{(t-1)^3}{120} + (t-1)^3\varepsilon(t-1)\right)}{\cancel{(t-1)^2}}$$

$$= \frac{1}{t-1} - 2 + \frac{t-1}{6} - \frac{(t-1)^2}{24} + \frac{(t-1)^3}{120} + (t-1)^3\varepsilon(t-1).$$